A STATISTICAL MODEL FOR INTELLECTUAL DEVELOPMENT

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1. Introduction

This study of individual intellectual development in the context of the family life cycle is part of a larger project aimed at re-examination and refining of existing models of the family as a social-psychological system. As such, it involves re-examination of the accepted "fundamental" variables and underlying principles in social psychology and social system theory. The present study is aimed towards developing new models of family and social systems. Such models should be structured in a manner so that the parameters which are inherent in the system would be "better" defined as random variables. Any conclusions which may be drawn from these systems will be greatly influenced by the behavior of their parameters. Also, the random variables should accurately describe the key aspects of the behavioral phenomena we are attempting to study, namely the behavior of individuals in their families and in small heterogeneous groups.

Along this line, a considerable amount of work has been expended in looking at the relationship between birth order of children and their later performance as adults (1), (3), (4), (6), (7), (8), (9), and (10). While this structural approach to the family might appear to be a rather superficial exercise in correlation, it has a certain amount of theoretical and methodological rationale. Birth order is a variable which has been used in theory construction, directly or indirectly, by sociologists, anthropologists, psychiatrists, and psychologists of many different schools of thought, covering several generations in the development of sociological and psychological concepts. These studies have drawn on clinical observations, field studies, and laboratory work. For example, in a current social psychology text (5), the question of birth order effects seems to be subsumed within the larger theme of investigating the effect of social system upon the development of individual differences such as personality, psychological capabilities, or adaptability.

Methodologically, the use of this type of variable avoids a great deal of the difficulty encountered in measuring the effects of hypothetical constructs such as motivation, defense mechanisms, or ego strength. Should the use of the easily observable birth order variable prove useful, it could mean a considerable savings of time and energy in laboratory and field research in social psychology, when compared to the problems involved in developing indirect measures of the other types of variables mentioned.

Unfortunately, the results of the birth order studies up to the present time have been contradictory and equivocal. Until recently, it was argued (8) that birth order effects not only have not been conclusively demonstrated, but also were not likely ever to be demonstrated because they represented oversimplified views of the real family system and were primarily interesting because they were methodologically easy to obtain. However, several recent studies (2) and (11) have dramatically changed that view. In our opinion, effects related to family structure seem to be emerging clearly.

One of the most dramatic of these recent birth order studies was that of Belmont and Marolla in 1973. As has often been the case previously, these effects were discovered as the by-product of a larger study - in this instance the influence of the Dutch famine of 1944-1945 upon longitudinal intellectual development. Their population cohort consisted of 386,114 Dutch males, virtually the entire population born or conceived during the famine, observed at age 19. The study shows what seems to be a clear-cut decline in intellectual development with increases in family size and birth order, although Belmont and Marolla did not offer an explanation for these factors.

In 1975, Zajonc and Markus (11) proposed what they called the Confluence Model to explain these structural effects. The first part of their efforts was directed toward obtaining a statistical description of the phenomena observed by Belmont and Marolla. This is used as a basis for formulating, in a mathematical sense, the idea that the level of intelligence of any individual is the result of the conflux of a number of developmental streams or trajectories primarily determined by his genetic endowment and familial environment. This would be modified into a new trajectory by the addition (or, logically, the loss) of members to the family, each with his or her own trajectory. Furthermore, each additional family member's potential development will be heavily influenced by the now changed environment at the time of birth.

The present paper outlines several simplifications of Zajonc and Markus' work and brings out some additional aspects of the model. The procedure for the analysis of the model is described in Section 2; the summary of the results of this study and the concluding remarks follow in Section 3.

2. Analysis of the Model

The emphasis of the present analysis is in two parts. First, there is a close re-investigation of the Dutch data to suggest improvements in the understanding of the effects of birth order and family size on intelligence. Secondly, we discuss a more realistic formulation and interpretation of the Confluence Model in view of the results of the first part.

The data reported by Belmont and Marolla consist of mean scores on the Raven Progressive Matrices intelligence test. These scores are broken down into 45 catagories ranging from only child (that is, first born in a one-child family) to ninth born in a nine-child family. The scores were linearly transformed (11) to standardize the mean score of only children at 100. We will denote the scores by M_{ij} ($i \leq j$; $i, j = 1, 2, \ldots, 9$), where i is the birth order (or rank) of a particular individual and j is the total number of siblings in that person's family (see Table 1).

Zajonc and Markus employ multiple regression techniques to regress the M_{ij} upon i, i^2, j and a variable termed the last born handicap, λ_{ij} ($\lambda_{ij} = 1 - \delta_{ij}$, where δ_{ij} is the usual Kronecker delta). They argued that the Raven scores, on the average, behaved parabolically (with upward concavity) with increasing birth order i. As can be seen in Figure 2, this would put the jth child in a j-child family in an anomalous position, hence the "handicap" λ_{ij}. We believe that this unnecessarily complicates the present statistical model, and does not, perhaps, reflect a true picture of the actual data. A different approach was taken, utilizing forward and backward stepwise regression, to regress the M_{ij} on i, i^2, i^3, j, and j^2. If the non-linear behavior seen by Zajonc and Markus were statistically significant, it would be reflected in a significant cubic term. In addition, Figure 2 suggests that the Raven scores might behave quadratically with increases in family size j. (The previous authors did not consider this.)

The fitted regression model was treated as a time series with respect to the variable i, or birth order, for fixed j. The auto and cross-correlation functions were examined.

Zajonc and Markus propose that the intelligence of an only child should develop according to the equation.

$$M(t) = \alpha(t)(1 - \exp(-k^2t^2)) \tag{2.1}$$

where M(t) is the transformed Raven score at time (or age) t, k is an arbitrary rate constant, and α is a family process variable which takes into account the effects of the individual's environment on his development. Postulating that the most dramatic changes would occur at significant times - that is, the birth of a later sibling - leads to a step function for the ith sibling in a j-child family.

$$M_{ij}(t) = \sum_{n=1}^{j} \alpha_{in}\{(1 - \exp(-k^2t^2_{n+1})) - (1 - \exp(k^2t^2_{n}))\} \tag{2.2}$$

where

$$t_n = \begin{cases} 0, & \text{if } n=i \\ \text{age of child i at birth of child n}, & \text{if } i<n\leq j \\ t, & \text{if } n=j+1. \end{cases}$$

Note that Equation 2.2 may be re-formulated to depend on time only through the age gaps between adjacent siblings and between the birth of the last child and time t. It was assumed (11) that the gap between all siblings but the last two is 2 years. The final gap was taken to be 4 years. The system of equations formed by 2.2 may be solved for the α_{ij}, using the transformed mean Raven scores, measured at t = 19. Since (as will be explained below) family size j is apparently a dominant variable in the process, the resulting coefficients were regressed upon family size.

3. Summary and Concluding Remarks

In summarizing the results of the present study, we shall follow the order of discussion of the previous section. The regression equation presented in (11) is certainly a good fit:

$$M_{ij} = 101.31 - .31i + .014i^2 - .37j + .48\lambda_{ij}; \tag{3.1}$$
$$r^2 = .97,$$

but some difficulties emerge. The last born handicap variable, λ_{ij}, was included to account for a deviation from an otherwise assumed pattern, namely that the data encompasses the vertex of a parabola with upward concavity, with respect to birth order i. However, it can readily be seen that the minimum of Equation 3.1 occurs at birth order 11, which is beyond the domain of the data. Thus the equation does not adequately reflect the apparent behavior Zajonc and Markus were trying to describe. Furthermore, we found that both forward and backward stepwise regression (using the variables i, i^2, i^3, j, and j^2) gave the same, simpler equation:

$$M_{ij} = 100.60 - .25i - .028j^2; \tag{3.2}$$
$$r^2 = .96.$$

The inclusion of more than two variables did not yield a significant improvement in the fit.

The simple correlation coefficient matrix suggests that the variable j would serve almost as well as j^2, see Table 2. Another regression with j forced into the stepwise procedure leads to the equation:

$$M_{ij} = 101.37 - .24i - .32j; \tag{3.3}$$
$$r^2 = .95.$$

The latter equation (3.3) may be preferred to Equation 3.2, because it is linear in both variables, birth order and family size, and there is no real loss in the goodness of fit of the model as measured by r^2. The residuals show no abnormal deviation of the last born data from the predicted scores; hence the last born handicap λ_{ij} does not appear as necessary for the model. If there is any trend at all, it would be that the scores of the (j-1)th sibling are slightly higher than predicted.

Our analysis agrees with the conclusion reached by Zajonc and Markus that family size has more influence than birth order in the process.

The inclusion of j or j^2 alone accounts for roughly 77-78% of the variance in Equations 3.2 and 3.3. The inclusion of i (in the presence of j or j^2) accounts for an additional 18%.

The sample auto-correlation function does not indicate stationarity in the process; that is, the nonhomogeneity in the mean effect seems to be further confirmed. The cross-correlation function shows strong evidence of a relationship between the variates M_{ij} for different family size, when the individuals in question have nearly the same birth order, but this decreases rapidly as the lag between birth orders increases. The cross-covariance reflects the same trend, but with large increases in variation with respect to larger family sizes.

After examining the Confluence Model and the rationale behind it, several refinements are evident from the nature of the data. The parameters α and k in Equations 2.1 and 2.2 should be regarded as stochastic variables. Zajonc and Markus note that the growth rate coefficient k will vary with the type of intellectual abilities being tested and the particular test used. It may also vary between individuals. It is not unreasonable to believe that at least part of these individual differences are due to random genetic patterns within a family.

Although not explicitly stated, it is implicit in the formulation of the model that the family process coefficient α is properly regarded as a random variable. It characterizes changes in the familial environment, such as birth order, family size, and other similar system components. In addition, since due to the form of Equation 2.1, α sets an asymptotic limit on the level of intelligence, part of the variation in α may be explained by genetic differences.

As mentioned previously, Zajonc and Markus solved the system described by Equation 2.2 for the α_{ij}. They then regressed these values on i, i^2, j, and λ_{ij}, and were able to account for 69% of the variance. Using their technique, the authors found that j alone will account for 60% of the variance in the α_{ij}, while j and j^2 accounts for 65%. This further confirms the relative importance of family size to the model over birth order. Also, it indicates that the inclusion of all the variables yields an equation with only a marginally better fit.

The authors are currently studying the behavior of Equations 2.1 and 2.2 in more detail. In their present formulation, the time dependency is reflected through the age gaps between the ith and (i+1)th sibling. resulting in step-function behavior for α as a function of time. The distribution of these gaps is being simulated by Monte Carlo techniques. One goal is to find an optimum pattern of age gaps to insure the highest average intelligence within a family, or the highest intelligence for a given sibling. Zajonc and Markus allude to regarding α as a continuous differentiable function of time, leading to a differential equation (although the latter is stated incorrectly). This would give the equation:

$$\frac{dM(t)}{dt} = \frac{d\ \ln\alpha(t)}{dt}M(t) + 2k^2t\{\alpha(t) - M(t)\}. \tag{3.4}$$

Coupled with a similar equation for $d\alpha(t)/dt$, which may be suggested by the simulation, the resulting stochastic differential system could be solved for the behavior of both M(t) and $\alpha(t)$.

In conclusion, the present study makes four contributions to the Confluence Model. First, the existing regression model can easily be simplified to give just as accurate a prediction of intelligence when family size and birth order are known. This involves detecting the strong linearity which exists in the data, and formulating the forwards and backwards stepwise regression. Secondly, some additional analysis gives a better insight into the behavior of the model. Thirdly, as is evident from the data which describes the system, the parameters α and k should be treated as stochastic variables. Finally, we believe that the present system ought to be characterized by a stochastic differential system (3.4). In the current form of the model, one can only define behavior of the system at discrete points in time (that is, birth of another child). We propose to investigate α and k as continuous random functions of time.

4. References

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TABLE 1

MEAN RAVEN SCORES BY BIRTH ORDER i AND FAMILY SIZE j

j \ i	1	2	3	4	5	6	7	8	9
1	100.00								
2	100.56	100.05							
3	100.60	100.12	99.69						
4	100.08	99.79	99.56	99.02					
5	99.72	99.38	98.95	99.02	98.36				
6	99.46	98.94	98.56	98.48	98.45	97.69			
7	98.99	98.53	98.14	98.27	98.17	98.16	97.10		
8	98.91	98.23	98.13	97.62	97.67	97.31	97.61	96.80	
9	98.12	97.98	97.45	97.29	96.98	96.83	96.56	96.89	96.26

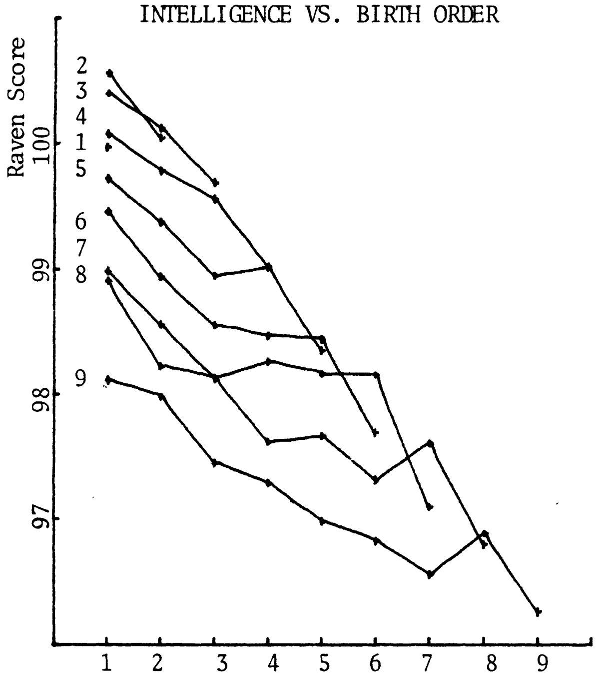
TABLE 2

CORRELATION MATRIX FOR M, i, i^2, i^3, j, and j^2

	M	i	i^2	i^3	j	j^2
M	1.000					
i	-.805	1.000				
i^2	-.771	.968	1.000			
i^3	-.716	.904	.981	1.000		
j	-.879	.500	.484	.452	1.000	
j^2	-.881	.491	.491	.471	.981	1.000

FIGURE 1

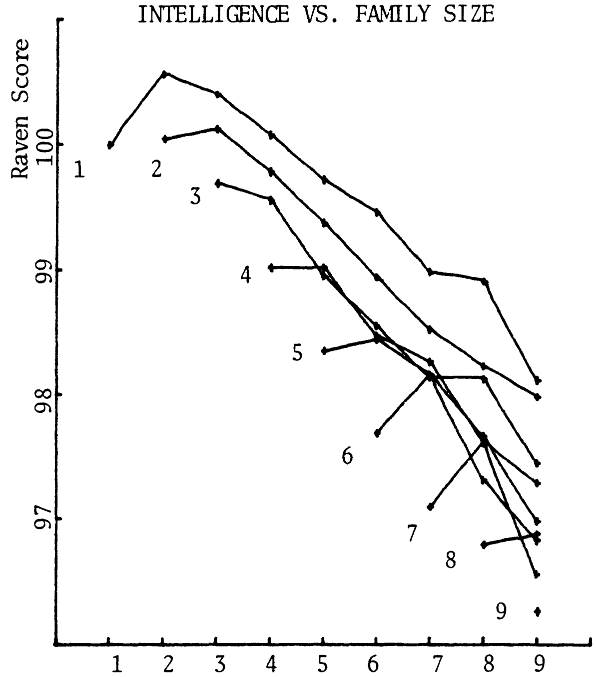
INTELLIGENCE VS. BIRTH ORDER



Note: Each curve represents a given family size.

FIGURE 2

INTELLIGENCE VS. FAMILY SIZE



Note: Each curve represents a given birth order.